CSEET
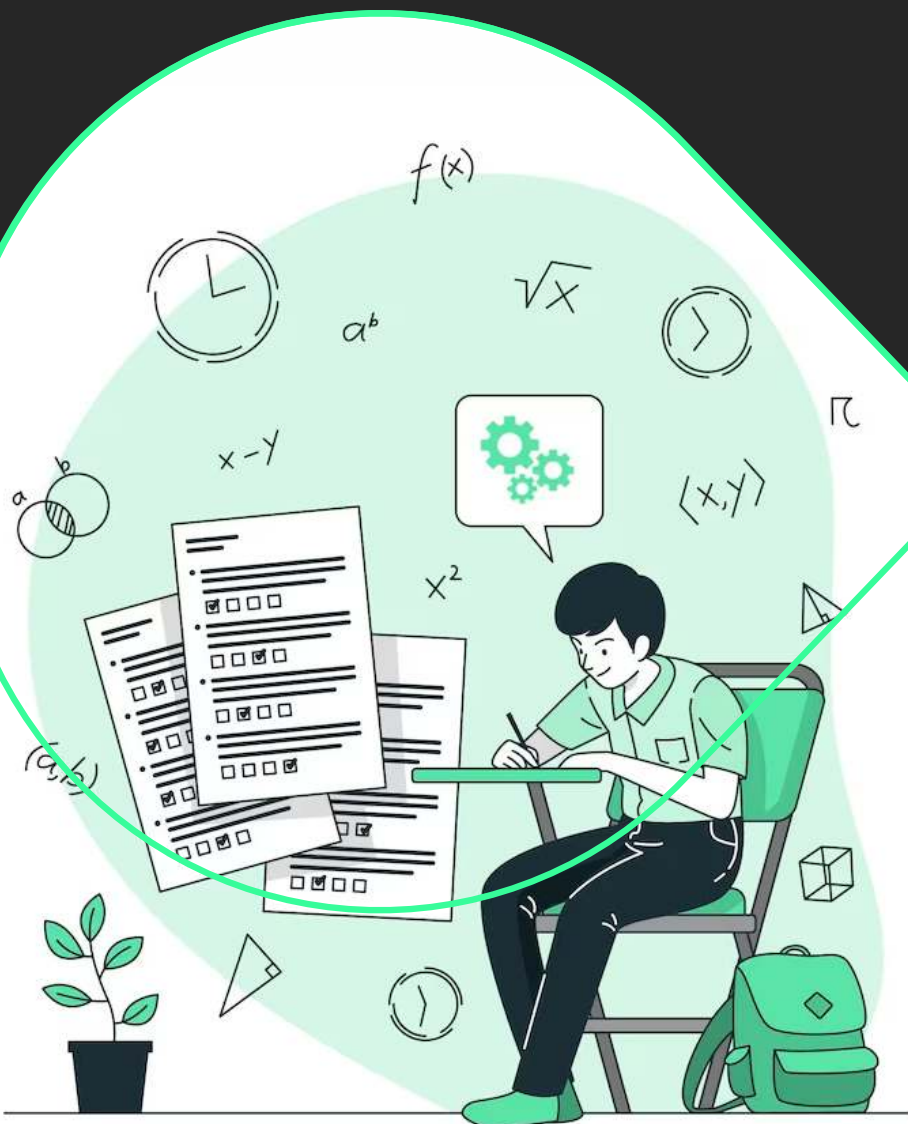# Quantitative Aptitude
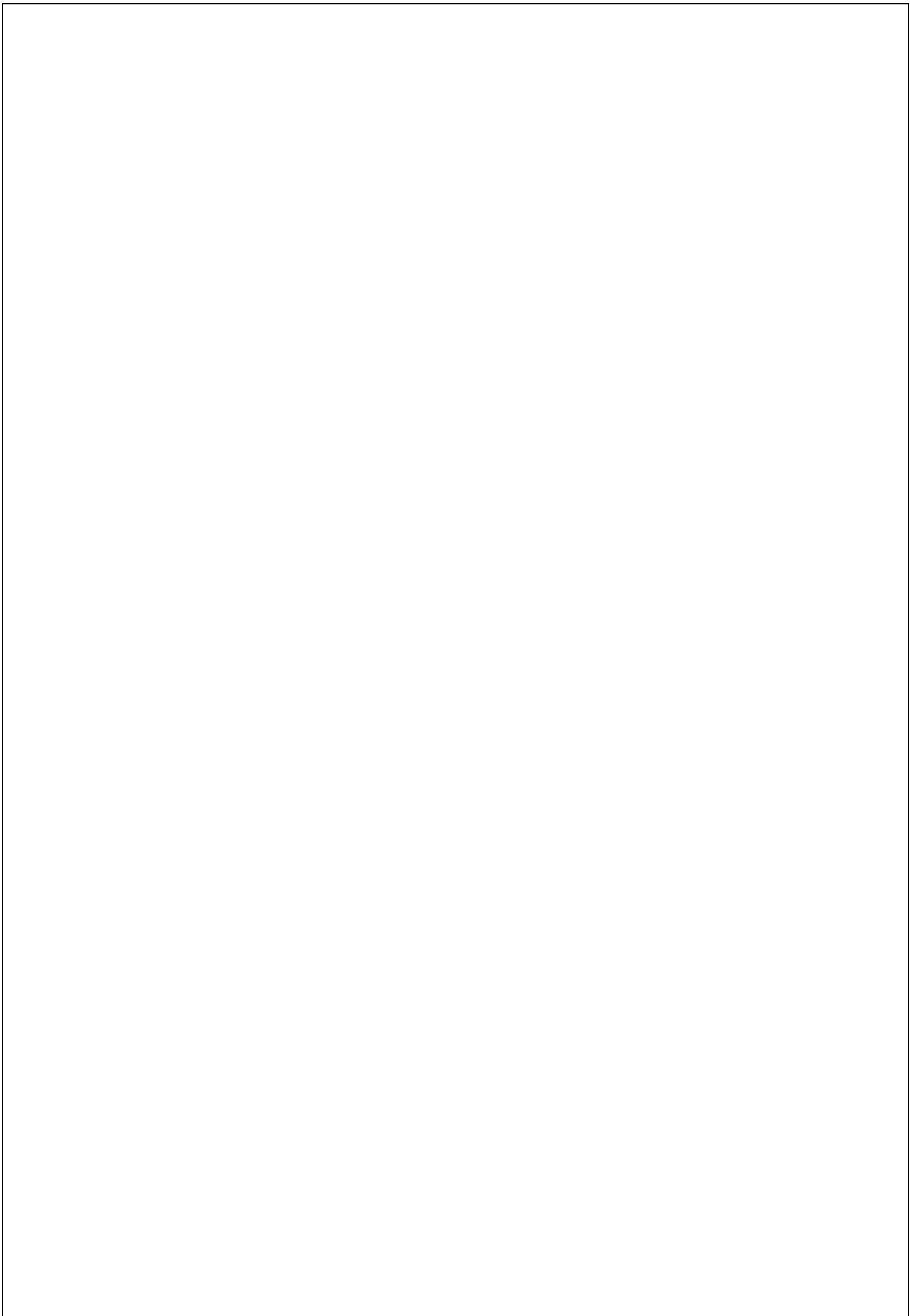
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Quantitative Aptitude (Maths)

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Number System

A number system is an arrangement of expressing the numbers in written form. Digits and Symbols in a consistent manner are used in number system. All the numbers are represented in the arithmetic and algebraic structure. The number system inter alia facilitates addition, subtraction, multiplication and division.

TYPES OF NUMBERS

The various types of numbers including the following:

1. Natural Numbers,
2. Whole Numbers,
3. Integers,
4. Rational Numbers,
5. Irrational Numbers,
6. Real Numbers and etc.

Let us discuss them in detail.

1. Natural Numbers

Natural numbers (N) are positive numbers i.e. 1, 2, 3 ..... and so one and so forth. Hence counting numbers in natural process like 1, 2, 3, ... constitute the system of natural numbers. These are the numbers which we use in our day-to-day life.

- It has to be noted there is no greatest natural number. For example, if 1 is added to any natural number, we get the next higher natural number, called its successor.
- Four-fundamental operations on natural numbers again generate natural number.

Four-fundamental operations of Natural Numbers are:

- Addition (Finding the Sum; '+') |
- Subtraction (Finding the difference; '-') |
- Multiplication (Finding the product; 'x') |
- Division (Finding the quotient; '÷')

- Addition and multiplication of natural numbers again yield a natural number; but
- subtraction and division of two natural numbers may or may not yield a natural number
- The natural numbers can be represented on a number line as shown below.
- Two natural numbers can be added and multiplied in any order and the result obtained is always same. This does not hold for subtraction and division of natural numbers.
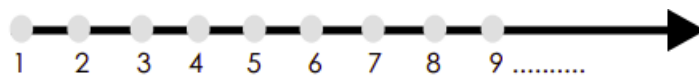
2. Whole Numbers

When a natural number is subtracted from itself, and one cannot say what is the left out number. To remove this difficulty, the natural numbers were extended by the number zero (0), to get what is called the system of whole numbers.

- Whole numbers do not include any fractions, negative numbers or decimals.
- Again, like before, there is no greatest whole number.
- The number 0 has the following properties:

$$a + 0 = a = 0 + a$$
$$a - 0 = a \text{ but } (0 - a) \text{ is not}$$
defined in whole numbers a
$$\times 0 = 0 = 0 \times a$$

- Division by zero (0) is not defined.
- Four fundamental operations can be performed on whole numbers also as in the case of natural numbers (with restrictions for subtraction and division).
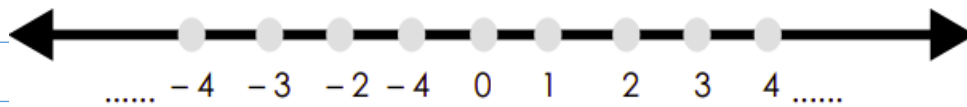- Whole numbers can also be represented on the number line as follows:
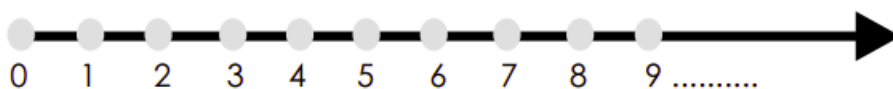
3. <u>Integers</u>

It is discovered that subtracting one number from another isn't always possible when working with natural numbers and whole numbers. For instance, in the system of natural numbers and whole numbers, the numbers (5 - 7), (6 - 14), (18 - 38), and so on are all impossible. As a result, it required an additional extension of numbers that permit such subtractions. As a result, it is necessary to multiply whole numbers by negative numbers like -1 (also known as negative 1), - 2 (also known as negative 2), and so on. $5 + (-5) = 0$, $10 + (-10) = 0$, $15 + (-15) = 0...$, $99 + (-99) = 0$, ..

As a result, we have expanded the whole number system to include another set of numbers known as integers. Therefore, the integers are: ..., − 7, − 6, − 5, − 4, − 3, − 2, − 1, 0, 1, 2, 3, 4, 5, 6, 7, ...

- It should be noted that although integers include negative numbers, they are equivalent to whole numbers in every way.
- Examples: -3, -2, -1, 0, 1, 2

Integers can be easily represented on the number line. For example, let us represent − 5, 7, − 2, − 3, 4 on the number line. In the below mentioned figure, the points A, B, C, D and E respectively represent − 5, 7, − 2, − 3 and 4.

> **Example 1:** Classify natural numbers, whole numbers and integers among the following:
>
> – 15, 22, – 6, 7, – 13, 0, 12, – 12, 13, – 31

> **Example 2:** Simplify the following and indicate whether or not the outcome is an integer. 12 × 4, 7/3, 18/3, 36/7, 14 × 2, 18/36, 13 × (–3)

Example 3: Using number line, add the following integers: (i) 9 + (– 5) (ii) (– 3) + (– 7)

4. **Rational Numbers**

   A number 'r' is called a rational number if it can be written in the form p/q, where p and q are integers and q ≠ 0.

- **When 'a' is a multiple of 'b'**

  Suppose a = mb, where m is a natural number or integer, then a/b = m.

- **When a is not a multiple of b**

  In this case a/b is not an integer, and hence is a new type of number. Such a number is called rational number

  Thus, a number which can be put in the form p/q, where p and q are integers and p/q, is called a rational number.

  Rational Numbers are of two types:

- **Positive Rational Numbers**

  A rational number p/q is said to be a positive rational number is p and q both are either positive integers or negative integers.

Thus 3/4; 5/6; -3/-2; -8/-6, -12/57 are all positive rational numbers.

- ## Negative Rational Number

  If the integers p and q are of different signs, then p/q is said to be negative rational number. For example, -1/2; 6/-5; -12/4 and 16/-3 are all negative rational numbers.

  > **Concluding Remark:**
  >
  > (i) Every natural number is a rational number but the vice-versa is not always true.
  >
  > (ii) Every whole number and integer are a rational number but vice-versa is not always true.

- ## Irrational Numbers

  From the previous discussion, we understand that there may be numbers on the number line that are not rational numbers. Hence, any number that cannot be expressed in the form of p/q, where p and q are integers and q ≠ 0, is an irrational number.

  Examples: √2, 1.010024563... , e, π

  We can also say that decimals numbers in any number line represent irrational numbers. Thus, a decimal expansion which is neither terminating nor is repeating represents an irrational number.

- ## Real Number

  Any number which can be represented on the number line is a Real Number(R). It includes both rational and irrational numbers. Every point on the number line represents a unique real number.

| TOPIC	DESCRIPTION
Natural Numbers	All counting numbers starting from 1,2,3,4,5......till infinity. The sum and multiplication product of two natural numbers is always a natural number; however, this doesn't stand are concerning subtraction and division.
Whole numbers	All counting numbers, including 0 (zero). These are also commonly called positive/non-negative integers. Like (0,1,2,3,4,5....)
Integers	The set of real numbers that consist of all-natural numbers, zero, and their additive inverses. (.....-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5.......)
Rational Numbers	All numbers that can be expressed as a ratio between two natural numbers in the form of fractions are called rational numbers. Like ½, ⅓, ¾, etc. All terminable decimals are also rational numbers.
Irrational numbers	Numbers can not be written as fractions, decimals, or ratios. For e.g. Square roots, unending decimals (0.33333333...etc.), pie, etc.
Real numbers	These are numbers that include all of the above types of numbers. Rational, irrational, natural numbers, whole numbers, and so and so forth

COMPUTATION OF WHOLE NUMBER

Given any natural number, one can add 1 to that number and get the next number i.e. one gets its successor. The successor of 16 is 16 + 1 = 17, that of 19 is 19 +1 = 20 and so on. The number 16 comes before 17, hence it is said that the predecessor of 17 is 17−1=16, the predecessor of 20 is 20 − 1 = 19, and so on.

Facts to be known for Whole Number

a. All positive integers, including 0 are whole numbers.

b. Real numbers are all whole numbers. .

c. Whole numbers make up all of nature's numbers. All natural numbers except 0 begin with 1. .

d. The smallest whole number is zero..

e. Natural numbers are regarded whole numbers, although fractions, decimals, and negative numbers are not. .

f. The number zero is the only one that has no sign..

g. Fractions are not included in whole numbers because, as the name suggests, a whole number is neither a fraction nor a decimal. The full number is not a fraction as a result, hence it cannot be negative.

h. Another name for counting numbers is whole numbers.

i. In mathematics, the numbers 0 through 1, 2, 3, and so forth stand in for the set of Whole Numbers.

j. The aforementioned facts demonstrate that all whole numbers and natural numbers are components of counting numbers. A whole number can also be obtained from the union of all positive counting integers plus zero.

k. A smallest whole number is 0 as it starts with zero (0).

l. The difference between the positive integer number line and negative integer number line is Zero.

Whole Number on Number line
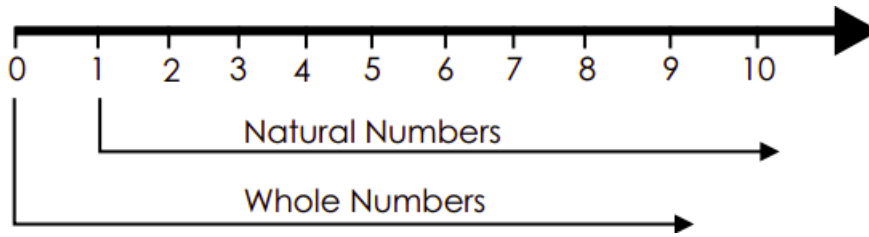
Whole Number Calculation Techniques

5. **Addition Property:** 0 does not change the final result. For Example- 2+0 = 2.

- **Closure Property:** Two Whole Numbers always produce a Whole Number as their product and their total. For example, 4 + 10 = 14 (A Whole Number), 4 * 10 = 40 (A Whole Number)

- **Associative Property:** The sum or product of the Whole Numbers remains the same regardless of how the numbers are organised. For example, 2 X 10 = 20 and 10 X 2 = 20 , 2 + 10 = 12 and 10 + 2 = 12, etc.

- **Multiplication Property:** The outcome of multiplying 1 by a whole number is that number itself. For example 7×1 = 7. If the whole number is multiplied by 0 then the result is 0. For example - 7×0 = 0.

- **Division Property:** When a whole number is divided by 0, the outcome is ambiguous. . For example- 7/0 = not defined.

- **Distributive Property:** This property is represented as P×(Q+R) = (P×Q) + (P×R). It is applicable for both addition and subtraction. For example - let P=11, Q=12, R=14, 11×(12+14) = (11×12)+(11×14) = 286.

- **Commutative Property:** P+Q = Q+P is a representation of this property. The property also holds for multiplication, but not for division or subtraction. For example - P=11, Q=12, 11+12 = 12+11 = 23.

Rounding of the Fractions

- Rounding off finds out the nearest whole number.
- For example: 7 is the closest whole number for 7.3.
- When the decimal number is less than .5, the whole number can be the number below the output.
- When the decimal point is .5 and above than, the whole number would be next whole number after rounding off.

SAMPLE QUESTIONS ON WHOLE NUMBERS

1. A number in which one-fifth part is increased by 20 is equal to one-tenth part is increase by 30. Find the number.
   a) 90
   b) 100
   c) 120
   d) 150

2. The Product of two numbers is 150 and the sum of squares of numbers is 325. Find the sum of both numbers.
   a) 24
   b) 25
   c) 29
   d) 30

3. Which is the largest four-digit number divisible by 91?
   a) 9919

b) 9900

c) 9909

d) None of these

4. If the number 61xx4 is divisible by 6, then what will be the value of x?

 a) 4

b) 5

c) 6

d) 7

5. Which of the following numbers is divisible by 13?

a) 1235

b) 1247

c) 1259

d) 1271

6. Write the smallest whole number.

7. What is the predecessor of whole number 0?

DECIMAL AND FRACTIONS

Key Points on Decimal and Fractions

- A fraction is a number representing a part of a whole.
- This whole may be a single object or a group of objects.
- A fraction whose numerator is less than the denominator is called a proper fraction, otherwise it is called an **improper fraction.**
- Numbers of the type 541 3 , 8 , 2 795 etc. are called mixed fractions (numbers)
- An improper fraction can be converted into a mixed fraction and vice versa.

- Fractions equivalent to a given fraction can be obtained by multiplying or dividing its numerator and denominator by a nonzero number .
- A fraction in which there is no common factor, except 1, in its numerator and denominator is called a fraction in the simplest or lowest form.
- Fractions with same denominators are called like fractions and if the denominators are different, then they are called unlike fractions.
- Fractions can be compared by converting them into like fractions and then arranging them in ascending or descending order.
- Addition (or subtraction) of like fractions can be done by adding (or subtracting) their numerators.
- Addition (or subtraction) of unlike fractions can be done by converting them into like fractions.
- Fractions with denominators 10,100, etc. can be written in a form, using a decimal point, called decimal numbers or decimals.
- Place value of the place immediately after the decimal point (i.e., tenth place) is 1/10, that of next place (i.e., hundredths place) is 1/100 and so on.
- Fractions can be converted into decimals by writing them in the form with denominators 10,100, and so on. Similarly, decimals can be converted into fractions by removing their decimal points and writing 10,100, etc. in the denominators, depending upon the number of decimal places in the decimals. Decimal numbers can be compared using the idea of place value and then can be arranged in ascending or descending order.
- Decimals can be added (or subtracted) by writing them with equal number of decimal places.

Convert Decimal to Fraction

To convert a Decimal to a Fraction, follow these steps:

- Step 1: Convert 0.50/ 1
- Step 2: Multiply each by 100

50/100

- Step 3: Simplify 50/100

  =1/2

1. Karan purchased 50 computers from a local computer market, only to discover that 10 of them were defective. Can you calculate the Fraction and Decimals of the defective computers in relation to the total computers purchased by Karan?

2. In an 100- office employees, 50 people chose burgers as a snack, while the other employees preferred mango juice. Calculate the Percentage of employees that choose a mango juice and give the result in Decimals.

3. Write 1/2th in Decimals.

4. If 3/2 of a number is 9, find the number.

5. Multiply 1.73 and 2.7.

6. Solve the following:

   (a) 4 – 1/3

   (b) 6 + 3/7

7. The product of two numbers is 1.178. If one of them is 049, find the other number.

8. 1/4 of a number equals 4/5 ÷ 1/10. What is the number? (NCERT Exemplar)

9. Simplify the following:

$$(i)\dfrac{2\frac{1}{2}+\frac{1}{5}}{2\frac{1}{2}\div\frac{1}{5}} \quad (ii)\dfrac{\frac{1}{4}+\frac{1}{5}}{1-\frac{3}{8}\times\frac{3}{5}}$$

DECIMAL FRACTIONS

Decimal fractions can be understood by considering normal fractions. A fraction has two parts numerator and denominator. It can be written as a/b.

Decimal fractions are those fraction in which denominator is 10, 100, 1000….. The numerator can be any number. These fractions are expressed in decimal numbers generally.

Examples of Decimal Fractions

- 9/100 can be expressed as 0.09.
- 180/100 can be expressed 1.80.
- 55/1000 is a decimal fraction written as 0.055.

Non-Examples of Decimal Fractions

Other fractions with non-ten numbers in the denominator are not decimal fractions. They are:

- 25/9
- 12/10125
- 91/125

Conversion to Decimal Fractions

- Let us consider an example of a fraction, 9/2.
- The first step would be to consider the number that gives 10 or a multiple of 10 when multiplied by the denominator. In this case, 5 multiplied by 2 gives 10.
- Now multiply the numerator and denominator with the same number to get your decimal fraction. Here, 9 × 5/ 2 × 5 gives 45/10.
- Thus, the decimal fraction of 9/2 is 45/10.

Conversion from decimal numbers to decimal fractions:

Write the original decimal number in the numerator and denominator form by placing 1 in the denominator: 9.5/1.

For every space that you move the decimal point, add a zero next to the 1 in the denominator:

95/10

9.5/1

95.0/10

Once the number in the numerator is non-decimal, you have got your decimal fraction: 9.5 = 95/10.

1. Convert 5x1/2 into a decimal fraction.
2. Convert 9.9 into a decimal fraction.

RELATIONSHIP BETWEEN NUMBERS

Cartesian Product of Sets

Suppose A is a set of 2 colours and B is a set of 3 objects, i.e.,

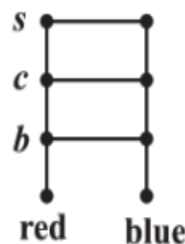   A = {red, blue}and B = {b, c, s},

where b, c and s represent a particular bag, coat and shirt, respectively. **How many pairs of coloured objects can be made from these two sets?**

Proceeding in a very orderly manner, we can see that there will be 6 distinct pairs as given below:

    (red, b), (red, c), (red, s), (blue, b), (blue, c), (blue, s).

Thus, we get 6 distinct objects (Fig 2.1).

From the illustration given above we note that

A × B = {(red, b), (red, c), (red, s), (blue, b), (blue, c), (blue, s)}.

Again, consider the two sets:

A = {DL, MP, KA}, where DL, MP, KA represent Delhi, Madhya Pradesh and Karnataka, respectively and

B = {01, 02, 03} representing codes for the license plates of vehicles issued by DL, MP and KA .

If the three states, Delhi, Madhya Pradesh and Karnataka were making codes for the licence plates of vehicles, with the restriction that the code begins with an element from set A, which are the pairs available from these sets and how many such pairs will there be?
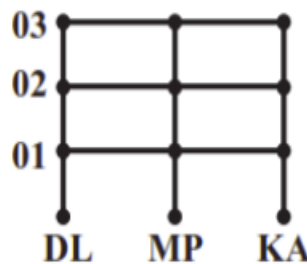
Fig 2.2

The available pairs are:(DL,01), (DL,02), (DL,03), (MP,01), (MP,02), (MP,03), (KA,01), (KA,02), (KA,03) and the product of set A and set B is given by

A × B = {(DL,01), (DL,02), (DL,03), (MP,01), (MP,02), (MP,03), (KA,01), (KA,02), (KA,03)}.

It can easily be seen that there will be 9 such pairs in the Cartesian product, since there are 3 elements in each of the sets A and B. This gives us 9 possible codes.

1. If $(x + 1, y - 2) = (3,1)$, find the values of x and y

2. If $P = \{a, b, c\}$ and $Q = \{r\}$, form the sets $P \times Q$ and $Q \times P$.

   Are these two products equal?

   Let $A = \{1,2,3\}$, $B = \{3,4\}$ and $C = \{4,5,6\}$. Find

   (i) $A \times (B \cap C)$

   (ii) $(A \times B) \cap (A \times C)$

   (iii) $A \times (B \cup C)$

   (iv) $(A \times B) \cup (A \times C)$

   Order relations, including strict orders:

   - Greater than
   - Greater than or equal to
   - Less than
   - Less than or equal to
   - Divides (evenly)
   - Subset of

   Equivalence relations:

   - Equality
   - Parallel with (for affine spaces)
   - Is in bijection with
   - Isomorphic

   Tolerance relation, a reflexive and symmetric relation:

   - Dependency relation, a finite tolerance relation
   - Independency relation, the complement of some dependency relation
   - Kinship relations

1. If half of one-third of a number is 20, then half-tenth of that number will be:

   1. 25
   2. 50
   3. 60
   4. 80

2. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 54. What is the difference between the two digits of that number?

   1. 2
   2. 5
   3. 6
   4. None of these

3. The difference between a two-digit number and the number obtained by interchanging the digits is 54. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 4:1 ?

   1. 4
   2. 8
   3. 16
   4. None of these

4. A two-digit number is such that the product of the digits is 18. When 63 is added to the number, then the digits are reversed. The number is:

   1. 19
   2. 29
   3. 39
   4. 49

5. The product of a two digit number is 14. When 18 is added to the number, then the digits interchange their places.
The number is:

(a) 68

(b) 59

(c) 95

(d) 86

6. If two fifth of one-third of a number is 50, then one-third of that number is:

(a) 376

(b) 3750

(c) 379

(d) 1250

FUNDAMENTAL ARITHMETICAL OPERATIONS

INTRODUCTION

According to Britannica, Arithmetic (a term derived from the Greek word arithmos, "number") refers generally to the elementary aspects of the theory of numbers, arts of mensuration (measurement), and numerical computation (that is, the processes of addition, subtraction, multiplication, division, raising to powers, and extraction of roots). The purpose of these operations is to simplify mathematical expressions. Basic mathematical operations are plus, minus, multiply and divide. These operations are helpful in daily life. It covers many a things touching every aspect including calculating Income and Expenditure to preparation of Balance Sheets.

The BODMAS rule is one of the most important rule while doing arithmetical operations. B stands for Bracket (), O stands for Order, D stands for divide (÷), M stands for multiply (×), A stands for addition (+), S stands for subtract (-).

BASIC ARITHMETIC OPERATIONS

Mathematical Operations

The four arithmetic operations make up the fundamental mathematical operations. The inverse of addition is subtraction, and vice versa. This means that if two numbers are joined together to get a third number. Than we can find the of the number added by subtracting the other number from the total.

Types of Fundamental Arithmetical Operation

Almost all forms of numbers, including integers, fractions, decimals, etc., can be subjected to arithmetic operations. Let's thoroughly comprehend each of the fundamental mathematical processes. The basic arithmetic operations in Mathematics are:

1. Addition (Finding the Sum; '+')
2. Subtraction (Finding the difference; '-')
3. Multiplication (Finding the product; '×')
4. Division (Finding the quotient; '÷')

Addition Rules

The following are the addition rules for integers:

- A positive integer is the result of adding two positive numbers.
- A negative integer is created by adding two negative integers.
- When subtracting positive and negative integers, utilize the sign of the biggest integer value.

Subtraction Rules

The following are the subtraction rules for integers:

- Both the numbers are (+), the answer will also be positive
- Both the numbers are (-), the answer will also be negativeIf one number is positive and other number is negative, the answer will be in sign which is largest

Multiplication Rules

The rules of multiplication are as follows:

- The product of two positive numbers is positive. In a multiplication if one number is positive and other number is negative, the answer will be negative.
- In a multiplication if both the numbers are negative, the answer will be positive.

Division Rules

- A positive integer is obtained by dividing two positive integers.
- A positive integer is obtained by dividing two negative integers.
- The negative integer is produced when you divide two integers with different signs.

Basic Arithmetic Properties

The basic arithmetic properties for real numbers are:

1. Commutative property
2. Associative property
3. Distributive property

6. Commutative Property

This property is applicable only for two arithmetic operations, i.e., addition and multiplication.

Suppose A and B are two numbers, then, according to commutative property –

| A+B = B+A | Example: 5 + 2 = 2 + 5 |
| A x B = B x A | Example: 1 x 5 = 5 x 1 |

Thus, the order of numbers in addition and multiplication does not change the result.

7. Associative Property

Like commutative property, the associative property is also applicable to addition and multiplication.

| A+(B+C) = (A+B)+C | Example: 1 + (5+3) = (1+5) +3 |

$A \times (B \times C) = (A \times B) \times C$	Example: $1 \times (5 \times 3) = (1 \times 5) \times 3$

Thus, if we change the grouping of numbers, the result does not change.

8. **Distributive Property**

According to the distributive property, if A, B and C are any three real numbers, then,

$A \times (B + C) = A \times B + A \times C$

Example: $5 \times (3 + 4) = (5 \times 3) + (5 \times 4)$

$5 \times 7 = 15 + 20$

$35 = 35$

Hence, proved.

SAMPLE QUESTIONS ON DMAS

1. Add 30 and 45 and then subtract 20 from the sum.

2. Solve: $10 + 10 + 10 + 10 + 10$.

3. Find the value of $(6 \times 4) \div 12 + 72 \div 8 - 9$.

4. Simplify: $24 - 4 \div 2 \times 3$

5. Simplify: $24 \div 4 \times 3 + 2$

6. Simplify: $(-20) + (-8) \div (-2) \times 3$

7. Simplify: $(-5) - (-48) \div (-16) + (-2) \times 6$

8. Simplify: $52 - (2 \times 6) + 17$

PERCENTAGE

As per Britannica Dictionary, Percentage, is a relative value indicating hundredth parts of any quantity. One percent (symbolized 1%) is a hundredth part; thus, 100 percent represents the entirety and 200 percent specifies twice the given quantity. For example, 1 percent of 1,000 Books equals 1/100 of 1,000, or 10 Books; 20 percent of the quantity is 20/100 1,000, or 200. These relationships may be generalized as x = PT/100 where T is the total reference quantity chosen to indicate 100 percent, and x is the quantity equivalent to a given percentage P of T. Thus, in the example for 1 percent of 1,000 Books, T is 1,000, P is 1, and x is found to be 10.

Percentage Formula

Percentage formula = (Value/Total value) × 100

Example: 3/5 × 100 = 0.6 × 100 = 60 per cent

Example: if only 10 of the 200 apples are bad, what percent is that?

As a fraction, 10/200 = 0.05

As a percentage it is:

10/200 x 100 = 5%

1. Calculate 20% of 80.
2. Calculate 25% of 100
3. 30% of 400 apples are bad. How many apples are bad?
4. A Mobile is reduced 25% in price. The old price was Rs. 120. Find the new price.

Variables of Percentage

Every percentage problem has three possible unknowns or variables :

- Percentage
- Part

- Base

In order to solve any percentage problem, you must be able to identify these variables.

Example: 80% of 20 is 16

- 80 is the percentage.
- 20 is the base.
- 16 is the part.

Increase or Decrease Percent

We often come across such information in our daily life as.

(i) 25% off on MRP

(ii) 10% hike in the price of Diesel.

% increase = [(New number – Original number)/Original number] x 100;

where, increase in number = New number – original number

% decrease = [(Original number – New number)/Original number] x 100

Where decrease in number = Original number – New number

1. The price of a laptop was INR 40,000 last year. It has increased by 25%this year. What is the price now?

Finding Discounts in Percentage

Discount is a reduction given on the Marked Price (MP) of the article.

So, Discount = Marked price – Sale price

1. A toy marked at Rs. 1,000 is sold for Rs. 920. What is the discount and discount %?
2. The market price of a Key Board is Rs. 300. A discount of 15% is announced on sales. What is the amount of discount on it and its sale price?

Percentage vis-à-vis Fraction and Decimals

Examples of percentages at par with fractions are:
1. 10% is equal to 1/10 fraction
2. 20% is equivalent to 1/5 fraction
3. 25% is equivalent to ¼ fraction
4. 50% is equivalent to ½ fraction
5. 75% is equivalent to ¾ fraction
6. 90% is equivalent to 9/10 fraction

SAMPLE QUESTIONS ON PERCENTAGE
1. If 20% of 30% of a number is 8, then find the number.

2. Which number is 30% less than 75?

3. The sum of (12% of 28.6) and (5% of 1.75) is equal to what value?

4. A stationery seller had some registers. He sells 40% registers and still has 420 registers . Originally, he had how many registers ?

RATIO AND PROPORTION

When two parameters are compared, a ratio is created by applying the division operator to the first and second values. The quotient x/y is typically referred to as the ratio between x and y when x and y are two parameters of the same type and with similar units, such as y is not equivalent to 0. The colon (:) symbol is used to denote ratios. It implies that the ratio x/y can be written as x: y and has no units. To put it another way, the ratio is the number used to represent one quantity as a fraction of the other item. Only if the two quantities in a ratio have the same unit can they be compared.

A proportion is a mathematical phrase that indicates the two ratios are comparable to one another. The similarity between the two fractional numbers or ratios is, to put it simply, the proportion. The two ratios are intended to be directly proportionate to one another when the two sets of specified quantities are changed in a similar way. The symbol (::) represents proportions and aids in figuring out ambiguous numbers.

Types of Proportion
There are two types of proportions as follows.

1. Direct Proportion
The term "direct proportion" describes the direct correlation of the two numbers. When one number rises, the other rises as well, and vice versa. For instance, if a vehicle's speed is raised, its distance traveled will undoubtedly increase.

2. Inverse Proportion
The term "inverse proportion" describes how two numbers are related in such a way that when one number rises, the other number falls, and vice versa. As a result, the

inverse ratio is written as a 1/b. For instance, if we drink more water from a bottle, there will be less water left in the bottle overall.

Ratio and Proportion Formula

The Ratio Formula is written as x : y ⇒ x/y where

x = Antecedent or the first term

y = Consequent or the second term

> For example, Ratio 8 : 4 is also written as 8/4, where 8 is called the antecedent and 4 is called the consequent.

In order to write a proportion in mathematics for the two ratios, a:b and y:z then we express it as a:b :: y:z → a/b = y/z

7. The two numbers namely b and y are called the **mean terms**.
8. The two numbers namely a and z are called the **extreme terms**.
9. In a : b = y : z, the numbers or parameters of a and b should be of the same type with similar units, while y and z may be the separate ratios of parameters of the same type with similar units. For example, 10 meter: 20 meter = 50 kg: 100 kg.
10. In the concept of proportion, the product of the mean terms is equivalent to the product of the extreme terms. Hence, we get b × y = a × z.
    For example, In the proportion of two ratios of 5 : 10 :: 10 : 20, we apply the formula of The Product of Mean Terms = The Product of Extreme Terms
    We get, 10 × 10 = 5 × 20 = 100
11. The proportion formula can be written in the form of a/b = c/d or a : b : : c : d.

Difference between Ratio and Proportion

Sr. No	Ratio	Proportion
(i)	When comparing various quantities with the same units, it is used.	It is used to describe a relationship between two ratios, each of which

		may have a different set of units. It is used to describe a relationship between two ratios, each of which may have a different set of units.
(ii)	To express a ratio, two symbols are used: a colon (:), and a slash (/).	It is possible to express a proportion using the double colon (::) symbol.
(iii)	It is defined as an expression.	It is termed as an equation.

Key Notes on Ratio and Proportion

By employing the idea of ratio, any numbers or parameters with comparable units can be compared. Only when two ratios are the same we can say that they are in a proportional relationship. A ratio always produces equivalent outcomes when the individual numbers are multiplied and divided by like numbers.

1. There are 63 students available in the 8th class. The number of students who want to study Sanskrit and the number of students who want to study Mathematics is expressed in the ratio 5:2. Calculate the number of students who want to study Sanskrit and those who want to study Mathematics.

2. R and S started an Electronic shop and decided to divide the profit between them in a ratio of 7:5. The total profit from that shop is Rs. 12,000 by the end of the financial year 2022. What will be the individual profit share for both R and S?

3. If Abhishek travels a distance of 25 km in 5 hours. How much distance can he travel in the time of 8 hours?

4. Calculate the numbers whose sum is 88 and they are written in the ratio of 4:4

SQUARE ROOTS

When an integer is multiplied by itself, the result is known as a square root. The result of a number multiplying itself is referred to as the square number. The symbol for a square root is a √ sign.

Square Root Definition

Any square root when multiplied by the same number, the result is the original number. For a perfect square number, we obtain perfect square roots.

Methods to Find Square Root of Numbers

12. To determine whether a number is a perfect square or an imperfect square, one must first determine the number's square root. A perfect square is defined as a number that can be expressed as the square of the number from the same number system.

13. Imperfect squares are those numbers whose square roots contain fractions or decimals.

Some of the key methods to find out the square root of a number are as follows:

1. Repeated Subtraction Method
2. Prime Factorization
3. Estimation Method
4. Long Division Method

Repeated Subtraction Method

One of the methods frequently used to determine the square root of a number is repeated subtraction. This approach involves repeatedly subtracting the perfect square number from subsequent odd integers, such as 3, 5, 7, 9, etc., until the result is zero. Starting with 1, the subtraction proceeds through 3, 5, 7, and so forth until 0 is reached. This approach counts how many times the value is deducted from one to get to zero. This count indicates the required square root of the given numbers.

36 − 1	35
35−3	32
32−5	27
27−7	20
20−9	11
11−11	0

The sum of the six subtraction operations is 0, as can be seen in the table above. Starting with 1, the subtraction continues until the odd number, 11, is reached. In total, 1, 3, 5, 7, 9 and 11 are deducted. This represents 6 occurrences. 6 is therefore the square root of 36.

Prime Factorization Method

The prime factorization method is a simple way to get a number's square root. By dividing the perfect square progressively, this approach divides it into its prime factors. The prime factor pairs are then paired. The square root of the perfect square is obtained by multiplying one element from each pair. Let us find the square root of 196.

The prime factorization of 196 = 2 × 2 × 7 × 7.

When we pair the prime factors and select one from each pair, we have 7 × 2 = 14. Hence, the square root of 196 is 14.

Estimation Method

An approximation method is the square root by estimation method. By making educated guesses about the values, this approach determines the square root of numbers. Taking 4 as an example, the square root is 2, while 9 is the square root, which is 3. Knowing that the square root of 5 will be between 2 and 3 is therefore simple. However, we will still have to check the value of $\sqrt{6}$ is nearer to 2 or 3.

Let us try finding out the square of 2.4 and 2.9.

The square of 2.4 = 5.76

The square of 2.9 = 8.41

Since the square of 2.4 is 5.76, which is approximately 6, we can say that the square root of 6 is approximately equal to 2.4.

Properties of Square Root

14. Only a perfect square number can have a perfect square root.

15. An even perfect square has an even square root.

16. The square root of an odd perfect square will be odd.

17. Because a perfect square cannot be negative, it is impossible to define the square root of a negative number.

18. A square root can be found for any number that ends in the digit of the unit, such as 1, 4, 5, 6, or 9.

19. It is impossible to obtain a perfect square root if the unit digit of an integer is 2, 3, 7, or 8.

20. A number cannot have a square root if it has an odd number of zeros at the end. Only an even number of zeros allows for the calculation of a square root.

Square Root

Formula

To determine the square root of a number, use the square root formula. The square root formula is $y = \sqrt{x}$ to make things easier. It is important to note that y x y=x. Here x is the square of a number y.

For e.g., $2 = \sqrt{4}$, where y = 2 and $\sqrt{x} = 4$, thus y x y= x, i.e. 2×2 = 4.

The square root of a perfect square integer like 16 is taken into account. Let's think about the square root of -16 now. The integer -16 has no true square root. $\sqrt{-16} = \sqrt{16} \times \sqrt{-1} = 4i$ (as, $\sqrt{-1} = i$)

Here, "i" is represented as the square root of –1. Hence, 4i is the square root of the number 16.

<u>SAMPLE QUESTIONS ON SQUARE ROOT</u>

1. Which of the following figures is a square in all its parts?
   a) 111
   b) 225
   c) 142
   d) 156

2. A perfect square number can never have the digit ..... at the units place.
   a) 1
   b) 4
   c) 8
   d) 9

3. 3. Evaluate $\sqrt{6241}$
   a) 72
   b) 75
   c) 78
   d) 79

4. Find the square root of 6724.
   a) 79
   b) 76
   c) 82
   d) 87

AVERAGE

In plain English, an average is a single number chosen to represent a group of numbers. This average is typically the arithmetic mean, which is the total of the numbers divided by the number of numbers in the group. The average of the numbers 2, 3, 4, 7, and 9 (which add up to 25) is, for instance, 5. An average could be another statistic like the median or mode depending on the situation. In mathematics, the central value of a set of data is expressed as the average of a list of data. It is defined mathematically as the ratio of the total number of data points to the number of units in the list.

Average = Sum of Values/ Number of values

Formula to Calculate Average

For a given set of variables, we can quickly calculate the average. Simply add up all the values, then divide the result by the total number of values. Average can be calculated using three simple steps. They are:

21. Step 1: Sum of Numbers:

Finding the sum of all the given numbers is the first step in calculating the average of a set of numbers.

22. Step 2: Number of Observations:

The next step is to determine how many numbers are there in the dataset.

23. Step 3: Average Calculation:

In order to arrive at the average, divide the total by the number of observations. Now, let us consider an example to calculate the average.

Arithmetic Mean

The most typical kind of average is called the arithmetic mean. The arithmetic mean is the sum of the as divided by n where n is a number. If n numbers are supplied, each number denoted by ai (where i = 1, 2,..., n), then:

Where,

24. n is the number of observations
25. i represent the index of summation l
26. and ai = data value for the given index

Geometric Mean

27.   By determining the nth root of the product of n numbers, the geometric mean is a technique for determining the central tendency of a set of numbers. In contrast to the arithmetic mean, which is calculated by adding the observations and then dividing the total by the number of observations, it is fundamentally different. However, in the case of the geometric mean, we first calculate the product of all observations before calculating the nth root of the product, assuming that n is the number of observations. The formula is given by – Geometric Mean,

28.   x1, x2, x3,…, xn are the individual items up to n terms

Harmonic Mean

The reciprocal of the average of the reciprocals of the given data values is referred to as the harmonic mean. The formula to find the harmonic mean is given by:

Harmonic Mean, HM = n / [(1/x1) + (1/x2) + (1/x3) +…+(1/xn)]

Where x1, x2, x3,…, xn are the individual items up to n terms.

Average of Negative Numbers

The procedure or formula to calculate the average is the same if the list contains any negative integers. Let's use an example to better grasp this. Example:

Find the average of 3, −7, 8, 12, −2.

Solution: The sum of these numbers

= 3 + (−7) + 8 + 12 + (−2)

= 3 − 7 + 8 + 12 − 2

= 14

Total Units = 5

Hence, average = 12/5 = 2.8

Questions on Averages

1. Find the average of 7, 3, 8, 9, 8
2. Find the average of 11, 13, 19, 22, 10
3. If the age of 9 boys in a team is 12, 13, 11, 12, 13, 12, 11, 12, 12. Then find the average age of boys in the team.
4. If the heights of females in a group are 5.1, 5.2, 5.6, 5.4, 5.9, 5.8, 5.10, 5.5, 6, 5.3. Then find the average height.

1.01

INTEREST (SIMPLE AND COMPOUND)

Interest is the extra sum that a borrower pays to a lender in addition to repaying the amount borrowed. For instance, a borrower might take out a loan for Rs. 10,000 and agree to pay an additional Rs. 100 as interest. The sum of interest received or paid over a predetermined period is known as an interest rate. The interest rate, for instance, would be 10% if the prior borrower agreed to pay the debt in full within a year.

Amount of interest you pay or earn depends on the following factors:
1. The rate of interest.
2. The amount of loan.
3. How long does it take to repay loans?

If you use a basic interest formula to determine your interest amount, an interest charge of Rs. 500 will be applied to a loan of Rs. 5000 with an annual interest rate of 10%.

Formula to Calculate Interest

Interest can be calculated using two methods. These two methods are:
1. Simple Interest
2. Compound Interest

- ### Simple Interest
  Simple Interest (SI) = P(Principal) × R(Rate of Interest) × T(Interest Period) / 100

  Here, P stands for the principal sum, R for the rate of interest, and T for the period of interest.

The total amount due in the end is made up of the principal plus the simple interest, or P + SI. For example,

1. An invested sum fetched a total interest of INR 10000 at the rate of 10% in one year. What was the original principal amount?

Simple Interest Formula for Months

The above formula can be used to determine simple interest on an annual basis. If P is the initial investment, R is the annual interest rate, and n is the duration (in months), then the following formula can be written:

$$\text{Simple Interest for 9 months} = (P \times 9 \times R) / (12 \times 100)$$

- **Compound Interest**

The formula for calculating the amount received when interest is compounded annually:

$$\text{Amount} = \text{Principal} (1 + \text{Rate}/100)$$

The total compounded interest over the term is calculated as

Compound Interest = Amount – Principal

Example

1. In how many years will an amount of Rs. 4000 will be doubled, if the interest rate is 10% per annum?

Difference between Simple Interest and Compound Interest

Compound interest is a different kind of interest. Simple interest is based on the principal amount, but compound interest is based on the principal amount along with interest over time. This is the main distinction between simple and compound interest. To further comprehend the idea of simple interest, let's look at a straightforward example.

SAMPLE QUESTIONS ON INTEREST

1. Arpit takes an Education loan of Rs 15000 from a bank for a period of 1 year. The rate of interest is 10% per annum. Find the interest and the amount he has to pay at the end of a year.

2. Ankur borrowed Rs 100,000 for 3 years at the rate of 4.5% per annum. Find the interest accumulated at the end of 3 years.

3. Find out the difference between the compound interests on Rs. 5 Lakh for 1 years at 9% per annum compounded quarterly and half-yearly?

PROFIT AND LOSS

Profit relates to gain; and **Loss** is the opposite of profit.

1. **Profit (P):** A product is sold at a profit if the price is higher than the cost price. For instance, if a piece of land was bought for Rs. 1,20,000 and sold for Rs. 2,20,000 four years later, there would be a profit of Rs 1 lakh.

2. **Loss (L):** When a product is sold for less than what it costs to produce, the seller suffers a loss. For instance, if a Laptop was purchased for Rs. 50,000 and sold for Rs. 35,000 a year later, the seller would have suffered a Rs 15,000 loss.

3. **Cost Price (CP):** It refers to the price at which a product is made or purchased. It can occasionally additionally cover overhead costs, transportation costs, etc. Shiva, for instance, paid Rs. 20,000 for an Air Conditioner and added Rs. 1500 for shipping and Rs. 2000 for installation. Therefore, the final cost price equals the amount of all completed expenditures, or Rs. 23,500. This cost price is divided into two more categories:
   a. Fixed Cost: Fixed cost is constant as it does not vary with situations.
   b. Variable Cost: It could vary depending on the situation.

4. **Selling Price (SP):** It's the price at which a product is offered for sale. It could be greater than, equal to, or lower than the item's cost price. For instance, if a store owner purchased a table for Rs. 800 and sold it for Rs 1000, the furniture's cost price is Rs 800 and its selling price is Rs 1000.

5. **Marked Price (MP):** Shop owners essentially label this to provide a discount to the customers in such a way that.,
   a. Discount = Marked Price – Selling Price
   b. Discount Percentage = (Discount/Marked price) x 100

6. **Profit Percent (P%):** It is the percentage of profit on the price on which the product was purchased or manufactured.

7. **Loss Percent (L%):** It is the percentage of profit on the price on which the product was purchased or manufactured..

Profit and Loss: Formulae

The profit or gain is equal to the selling price(SP) (-) cost price(CP).

Loss is equal to the cost price (CP)(-) selling price(SP). The formula for the profit and loss percentage is:

Profit percentage (P%) = (Profit /Cost Price) x 100

Loss percentage (L%) = (Loss / Cost price) x 100

Profit	Loss
Cost Price (CP) < Selling Price (SP)	Cost Price (CP) > Selling Price (SP)
Profit= S.P.-C.P.	Loss= C.P.-S.P.
S.P. = C.P. + Profit	C.P. = S.P. + Loss
C.P. = S.P. - Profit	S.P. = C.P. – Loss
Profit % = Profit\C.P × 100	Loss % = Loss/C.P × 100

QUESTIONS ON PROFIT AND LOSS

1. Suppose a shopkeeper has bought 1 kg of Mangoes for Rs. 120. And sold it for Rs. 150 per kg. How much is the profit earned by him?

2. Calculate the percentage of the profit gained by the shopkeeper in above situation.

3. A man buys a Cooler for Rs. 2000 and sold it at a loss of 15%. What is the selling price of the Cooler ?

4. A Computer is sold at Rs. 12,050 with 15% profit. What would be the gain or loss percentage if it had been sold at Rs. 10,980?

5. Suppose the CP of 25 pencils is the same as the SP of some pencils . If the profit is 20%, then what is the number of pens sold?

6. A dealer sells goods at a 9% loss on cost price but uses 20% less weight. Compute profit or loss percentage.

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"the only impediment in the path of success is a person's own mindset; if that is controlled, every feat is achievable"

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