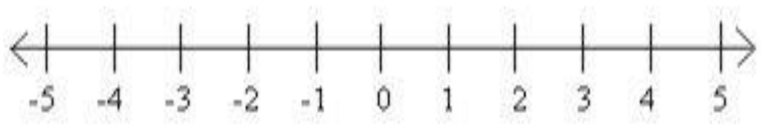
NUMBER SYSTEM

In this first topic 'Number System', we deal with 'Numbers' themselves. What are numbers? Where do we see and meet them? Are they of different types? Can we learn these distinct types of numbers?

Number line: A number line is line where all the numbers are allocated their positions. The origin of the number line starts from zero and it continues to infinity, on either side.



Few Basic Types of Numbers -

Positive Numbers : Numbers which are to the right of zero are said to be positive numbers. For example 1, 3 ,1.2 , 2.6 , 7 etc.

Negative Numbers : Numbers which are to the left of zero are said to be negative numbers. For example -1, -5 , -7.2 , -2.5 , -9 etc.

Counting Numbers: Counting numbers are those numbers which are well managed on the number line with the difference of 1. The smallest counting number on the number line is 1.

Natural Numbers: Numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11...and so on are called natural numbers. They are also called positive integers. Also we can say that the other name for counting numbers is natural numbers. The lowest natural number is 1.

Whole Numbers: Whole numbers are numbers without fractions, they take integral values. Whole numbers are those numbers which start from 0 or we can say if 0 is included in set of counting numbers (natural numbers) then we get set of whole numbers.Remember whole numbers would always take non- negative integral values

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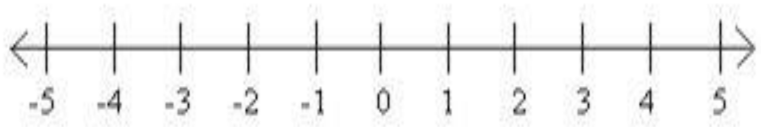
Integers: It is combination of both positive and negative numbers lying on the number line including zero. Remember zero is an integer. The integers are the natural numbers, their negatives, and the number zero like …,-3,-2,-1,0,1,2,3,… and they go on forever in both directions of the number line.



Rational numbers: Rational numbers are those, which can be written in the form of a ratio of x/ y, where the denominator y is non-zero.

Irrational numbers: Numbers, which are non-terminating and non- recurring (non-repeating) decimals are said to be irrational or we can say irrational numbers are those, which are not rational, that is those numbers that cannot be written in the form of a ratio x/y. For example = 1.414213….. , $\sqrt{3}$ =1.732050…..

Fractions: Fractions are those numbers, which are in the form of p/q where q is non-zero. For example 4/5, 5/6, 6/7 etc.

Real Numbers: Any number which can be plotted on the number line is a real number .The number can be positive or negative in nature. For example it may be like as 3, 4, 5, 6, -6, -5, -4, -3, -2…..

Prime numbers: The numbers, which have exactly two factors, 1 and the number itself, are called prime numbers.There are 25 prime numbers between 1 and 100.

Composite number: The composite numbers are those numbers which can be written as the product of prime numbers in a unique way.

Example 4, 6, 8,…

Note that composite number always has more than two factors and prime number cannot be a composite number.

Rounding numbers: When the number is approximated to the nearest possible integral value to maintain the accuracy of the data then the number is called rounded.

Properties of Numbers

The following is a handy list of tips that you can remember about numbers (think about each one of these):

- The number line goes on till infinity in both directions, which is indicated by the arrows.
- The numbers on the number line are indicated by their respective signs, which shows that the line includes both positive and negative numbers.
- The integer zero is neutral and neither positive or negative.
- The number in the halfway of 1 and 2 is 1.5 and in the half way of -1 and -2 is -1.5.
- Numbers, which are terminating and non-recurring are rational numbers. Similarly numbers, which are non-terminating and recurring are rational numbers.
- Set of natural numbers is contained in set of integers which is contained in set of rational numbers which is further contained in set of real numbers, which is further contained in set of complex numbers.
- Addition as well as product of two real numbers is a real number.
- Two real numbers can be added or multiplied in either order i.e. Addition and multiplication of real numbers is commutative.
- Two real numbers cannot be subtracted or divided in either order i.e. Subtraction and Division of real numbers is not commutative.
- In set of real numbers we don't define square root of negative numbers.

Properties of Zero

- a x 0 =0 always for any real number a.
- a +0 = a always for any real number a.

- $a - 0 = a$ always and $0 - a = -a$ for any real number a.
- $0/a = 0$ when "a" is a non-zero real number.
- $a/0$ is not defined i.e. we don't define division by zero.
- $a^0=1$ for any non-zero real number a.
- 0^0 is not defined.

Properties of Prime Numbers

- If *p* is any prime number and *p* divides a product of two integers say *m* and *n* i.e. p|mn (read as p divides mn), then *p* divides '*m*' or *p* divides '*n*' or both.
- Number of prime numbers is uncountable. i.e. there are infinite prime numbers.
- Every Prime number has exactly two factors or divisors.
  For example: 13 is prime number as the divisors of 13 are 1 and 13.
- There is only one even prime i.e. 2 and all other primes are odd.
- G.C.D of prime numbers is always 1. The numbers whose G.C.D is 1 are said to be co-prime thus we say two primes are always co-prime.
- Two numbers are co-prime if their HCF is 1. For example G.C.D. (21, 25)=1 and hence 21 and 25 are co- prime.
- The Twin Primes are pair of primes of the form (p, p+2).
- The term odd prime refers to any prime number greater than 2. For example: 3, 5,7,.............
- 1 is neither prime nor composite number.

How to check whether a number is prime or not?

We need to follow the following steps to find out if any given number is a prime number -

Step 1: Find square root of N, call it as K (Just find approximate values)

Step 2: Write down all the prime numbers less than K.

Step 3: Check divisibility of N with these prime numbers, which we have got in Step 2.

Step 4: If N is not divisible by any of the prime numbers then N is prime.

Example:

Let us check whether 211 is prime or not?

Solution:

Step 1: We find square root of 211 i.e. $K=\sqrt{211} = 14.52$

Step 2: We write all primes less than 14.52 i.e. 2, 3, 5, 7, 11 and 13.

Step 3: Since 211 is not divisible by any of these prime numbers, hence 211 is a prime number.

Example:

Let us check whether 313 is prime or not?

Divisibility Rules

A divisibility rule is a short cut for discovering whether a given number is divisible by a fixed divisor without performing the division, usually by examining its digits. Most of the divisibility rules are derived from a concept of remainders.

The two basic rules for finding out whether the number is divisible or not by any particular number:

1. If the divisor is prime number, then we can directly check for it.
2. If the divisor is a composite number then do factorization of divisor into factors, which are co-prime and check the divisibility for each prime factor individually.

DIVISIBILITY RULES: 2 TO 7

Divisibility Rule	Condition Check	Examples: To check
2	Last digit should be multiple of 2 or number is in form of 2n.	64 is divisible by 2 93 is not All Number ending with 0 2 4 6 8
3	Number is divisible by 3 if the sum of all the digits of number is divisible by 3	297156 is divisible as 2+9+7+1+5+6= 30 and 30 is divisible by 3
4	Last two digit for 2^2= 4. General form for 2 4 8 16 is to find the same last number of digit as the power of 2^n.	124 is divisible by 4 as for 4 divisibility will be check for 2^2 <= last 2 digits. Hence 24 in 124 are divisible by 4. Similarly 196 2048 are divisible by 4
5	The last digit ending with 0 or 5	1245685 is divisible by 5 because 5 is the last digit. Similarly 120 29650 12465 all are divisible by 5
6	Now for any number which have prime factors so check the divisibility for all the prime factors it has. For example 6 have 2 prime factors i.e. 2 & 3	4092 is divisible by 6 as by checking the divisibility rule for 6=2 X 3. 4098 is divisible by both 2 & 3. Whereas 196 which is divisible by 2 but not 3 hence it is not divisible by 6
7	**Check 1** : Subtract 2 times the last digit from the rest of the number **Check 2** : Make alternative sum of blocks of three from right to left	1: 553 is divisible? Double 2 is 6, 55-6 = 49 hence divisible by 7 2: 2147747 are divisible? = yes Alternative blocks 747 – 147+ 2= 602 and 602 are divisible by 7.

DIVISIBILITY RULES: 8 TO 12

Divisibility Rule	Condition Check	Examples: To check
8	For a number in form of 2^n count the last n digits. So for 8 it will 2^3 that means last 3 digits should be divisible by 8 or should be 000	178512 is divisible? Yes because last 3 digits that is 512 is divisible by 8
9	Any number is divisible by 9 if the sum of digits of number is divisible by 9	16291827 will sum as = 1+6+2+9+1+8+2+7=36 which is divisible by 9
10	Any number ending with last digit as ZERO 0 is divisible by 10	15210 has last digit as 0 hence divisible by 10
11	To check whether the number is divisible by 11 or not. form the alternating sum of the digits such that +-+-+-= final sum should be either zero or divisible by 11	153248799 = 1-5+3-2+4-8+7-9+9 = 0 hence number is divisible by 11 215784624 = 2-1+5-7+8-4+6-2+4= 11 again divisible by 11
12	12 = 3 X 4 or 3^1 X 2^2. So to check the divisibility of 12. The number should satisfy both the condition it should be divisible by 3 and also it should be divisible by 4	For 158496 check Divisibility of 3 = sum of digits should divisible by 3 = hence 1+5+8+4+9+6= 33 which is divisible by 3. Divisibility of 4 = number last two digits should be divisible by 4 = which in this case is 96 hence it is divisible by 4 Hence the number is divisible by 12

DIVISIBILITY RULES: SPECIAL NUMBERS

Divisibility Rule	Condition Check	Examples: To check
16	16 is nothing but = 2^4 Hence divisibility check for 16 is to check whether the number last 4 digits are either divisible by 16 or 0000	**20000** is divisible by 16 because last 4 digits are 0000 Similarly 19680 is also divisible by 16 as last 4 digits are divisible by 16
27	27 is cube of 3 but a number is divisible by 27 only if the sum of block of 3 from right to left is divisible by 27	828279 sum of block of 3 will be = 828+279= 1107 which is divisible by 27 hence the number is also divisible by 27
50	A number is divisible by 50 if the last two digits of number is either 00 or 50	For example 100 , 1050 both are divisible by 50
99	To check the number is divisible by 99 1: Check whether the number is divisible by 11 & 9 individually 2. Sum all the digits of number by making block starting from right to left	21285 : Divisible by 9 because sum of digits is 18 Divisible by 11 as 2-1+2-8+5 =0 Hence number is divisible by 99

Example 1:

Check divisibility of 124 by 2, 3, 4 and 5.

Solution:

124 is divisible by 2 since the last digit is 4.

124 is not divisible by 3 since the sum of the digits is 7 (1+2+4 = 7), and 7 is not divisible by 3.

124 is divisible by 4 since 24 is divisible by 4.

124 is not divisible by 5 since the last digit is 4 it is neither 0 nor 5.

124 is divisible by 2 and 4 not by 3 and 5.

Example 2:

Check divisibility of 225 by 2, 3, 4, 5, 6, 9 and 10.

Solution:

225 is not divisible by 2 since the last digit is not 0, 2, 4, 6 or 8.

225 is divisible by 3 since the sum of the digits is 9 and 9 is divisible by 3.

225 is not divisible by 4 since 25 is not divisible by 4.

225 is divisible by 5 since the last digit is 5.

225 is not divisible by 6 since it is not divisible by 2.

225 is divisible by 9 since the sum of the digits is 9.

225 is not divisible by 10 since the last digit is not 0.

225 is divisible by 3, 5, 9.

Example 3:

Check divisibility of 400 by 2, 3, 4, 5, 6, 8, 9 and 10.

Solution:

400 is divisible by 2 since the last digit is 0.

400 is not divisible by 3 since the sum of the digits is 4 and 4 is not divisible by 3.

400 is divisible by 4 since 00 is divisible by 4.

400 is divisible by 5 since the last digit is 0.

400 is not divisible by 6 since it is not divisible by 3.

400 is divisible by 8 since the last 3 digits are 400 and 400 is divisible by 8.

400 is not divisible by 9 since the sum of the digits is 4 and 4 is not divisible by 9.

400 is divisible by 10 since the last digit is 0.

400 is divisible by 2,4,5,8 and 10.

Tricks for Divisibility

$a^n - b^n$ is always divisible by a-b

$8^5 - 5^5$ is divisible by 8-5= 3

Remember it by:

$a^3 - b^3$ is divisible by a-b

$a^2 - b^2$ is also divisible by a-b

$a^n - b^n$ is divisible by a+b when n is even

$7^{10} - 5^{10}$ is divisible by 7-5= 2

Remember it by:

$a^3 - b^3$ is not divisible by a+b

$a^2 - b^2$ is divisible by a+b

$a^4 - b^4$ is also divisible by a+b

$a^n + b^n$ is divisible by a+b when n is odd

$7^{11} + 5^{11}$ is divisible by 7+5= 12

Remember it by:

$a^3 + b^3$ is divisible by a+b

$a^2 + b^2$is NOT divisible by a+b

$a^4 + b^4$ is NOT divisible by a+b

$a^n + b^n + c^n$ is divisible by $a+b+c$ when n is odd.

$7^3 + 5^3 + 2^3 = 343+125+8=476$ divisible by $7+5+2=14$

Example 1: $32^{23} + 17^{23}$ is definitely divisible by....

a. 49

b. 15

c. 49 & 15

d. none of these.

Example 2: $32^{23} - 17^{23}$ is definitely divisible by....

a. 49

b. 15

c. 49 & 15

d. none of these.

Example 3: $32^{232} - 17^{232}$ is definitely divisible by....

a. 49

b. 15

c. 49 & 15

d. none of these.

Example 4: $3^5 + 5^5 + 7^5$ is definitely divisible by....

a. 8

b. 7

c. 15

d. all of these.

Least Common Multiple (LCM)

Method to calculate LCM

Division method is a method to find the L.C.M. of numbers.

1. First we will write the numbers in ascending order.
2. Now we will divide the numbers with a common prime factor and continue the same till the time it is possible.
3. We multiply all common prime factors and numbers obtained in last row to get the LCM.

Remember if a number is not divisible by this prime number, then write the number as it is

Example : Find the L.C.M. of 24, 18 and 36?

Example 2: Find the L.C.M. of 60, 16 and 48?

Factorial

A factorial is a non-negative number which is equal to the multiplication of numbers that are less than that number and the number itself. It is denoted by (!)

Let's take an example to understand this

What will be the value of 5!

So in the above definition we discussed that the multiplication of the numbers which all are less than that number and the number itself. Hence number less than 5 are 1,2,3,4 and 5 is number itself so

5! = 5 x 4 x 3 x 2 x 1 = 120

Always remember we define the value of 0! =1

Value of 6! = 6 x 5 x 4 x 3 x 2 x 1 = 720

Value of 4! = 4 x 3 x 2 x 1 = 24

Value of 3! = 3 x 2x 1= 6

We can also write n! = n x (n – 1)!

Type 1: Highest power of p (p is a prime number) which divides the q!

Example: What is the highest power of 3 that divides 13! ?

Solution:

13! = 13 x 12 x 11 x 10 x 9 x 8 x7 x 6 x 5 x 4 x 3 x 2 x 1

And we have to find highest power of 3 that can divide the above term

So first we need to know how many times 3 is multiplied in 13! .

So to find the number of 3's we will divide 13 with 3.

13 divided by 3 gives 4 as quotient and 1 remainder. We will keep remainder aside and move further till the quotient cannot be divided further

Again we divide 4 by 3 we get quotient as 1 and again remainder is 1 .Now we stop at this stage because quotient 1 cannot be divided by 4.

Now add all the quotients

4+1 =5.

So the maximum power of 3 is 5, which can divide the 13! .

Example: What is the highest power of 7 that exactly divides 49!

Solution: To find the highest power of 7 that exactly divides 49! . We need to know the number of 7's in the 49!

So when we divide 49 with 7, the quotient will be 7 and there is no remainder, since the quotient can further divided by 7 we will divide 7 with 7 with quotient 1 and remainder 0 .

Now add all quotients and get the answer as 8

So the highest power which will divide the 49! will be 8.

Type: Highest power of p which divides the q! ,where p is not a prime number

Example 1

What will be the maximum power of 6 that divides the 9!

In order to find maximum power of 6 we will first write as product of 2 and 3.

Example 2

What will be the highest power of 12 that can exactly divide 32!

We can write 12 = 2x2x3 i.e. we need pair of 2^2 x 3

How to find the Unit Digit of a number

For the concept of identifying the unit digit, we have to first familiarize with the concept of cyclicity. Cyclicity of any number is about the last digit and how they appear in a certain defined manner. Let's take an example to clear this thing:

The cyclicity chart of 2 is:

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

You would see that as 2 is multiplied every-time with its own self, the last digit changes. On the 4^{th} multiplication, 2^5 has the same unit digit as 2^1. This shows us the cyclicity of 2 is 4, that is after every fourth multiplication, the unit digit will be two.

Cyclicity table:

The cyclicity table for numbers is given as below:

Number	Cyclicity
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1

What is the unit digit of the expression 4^{993}?

Now we have two methods to solve this but we choose the best way to solve it i.e. through cyclicity

We know the cyclicity of 4 is 2

Have a look:

$4^1 = 4$

$4^2 = 16$

$4^3 = 64$

$4^4 = 256$

From above it is clear that the cyclicity of 4 is 2. Now with the cyclicity number i.e. with 2 divide the given power i.e. 993 by 2 what will be the remainder the remainder will be 1 so the answer when 4 raised to the power one is 4. So the unit digit in this case is 4.

Note : If the remainder becomes zero in any case then the unit digit will be the last digit of $a^{\text{cyclicity number}}$

where a is the given number and cyclicity number is shown in above figure.

The digit in the unit place of the number 7^{295} X 3^{158} is

A. 7

B. 2

C. 6

D. 4

Solution

The Cyclicity table for 7 is as follows:

$7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2401$

Let's divide 295 by 4 and the remainder is 3.

Thus, the last digit of 7^{295} is equal to the last digit of 7^3 i.e. 3.

The Cyclicity table for 3 is as follows:

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

Let's divide 158 by 4, the remainder is 2. Hence the last digit will be 9.

Therefore, unit's digit of (7^{925} X 3^{158}) is unit's digit of product of digit at unit's place of 7^{925} and 3^{158} = 3 * 9 = 27. Hence option 1 is the answer.